Y. A. GOVT. DEGREE COLLEGE FOR WOMEN
CHIRALA, BAPATLA (DIST.)

NAAC - B

DEPARTMENT OFMATHEMATICS

PROJECT WORK

ON

"NUMERICAL ANALYSIS ADVANCED NUMERICAL ANALYSIS INTEGRAL TRANSFORMATIONS"

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This is submitted to ACHARYA NAGARJUNA UNIVERSITY in partial fulfillment as for the requirement for the Degree in Bachelor of Science in Mathematics

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CERTIFICATE

This is to certify that the main project entitled Numerical Analysis, Advanced Numerical Analysis, Integral Transformation is being submitted by the following students:

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CONTENT

NUMERICAL ANALICIS

UNIT-2

Solutions of Algebric and Transcendal Equations

SOLUTION OF ALGBRAIC AND TRANSCENDENTAL EQUATIONS

INTRODUCTION:-

The Equation $f(x)=0$ is said to be algeboric it $f(x)$ is purely a polynomial in x.

The equation $f(x)=0$ is said to be transcendental it is not algeboraic an Equation. But it $f(x)$ involves trigonometrical arithmetic or Exponential terms in it, then it is called transcendental equation.

The Solutions of on algeboric Equation is the process of finding a number or set of numbers that it is substituted for the varibales in the Equation, reduce it to on identity.

Solution of Algebraic and Transcendental equations.

Iterative Methods [Bisection Method]

Statement of the problem

Find a root of the equation $F(x) = x^3 - 4x - 9 = 0$ using bisection Method

Solution :-

Aim :-

A root of the equation $F(x) = x^3 - 4x - 9 = 0$

Formula :-

$$x_1 = \frac{a+b}{2}$$

procedure :-

Given equation $F(x) = x^3 - 4x - 9 = 0$

$$F(1) = 1^3 - 4(1) - 9$$

$$= 1 - 4 - 9 = -12 < 0$$

$$F(2) = (2)^3 - 4(2) - 9$$

$$= 8 - 8 - 9 = -9 < 0$$

$$F(3) = (3)^3 - 4(3) - 9$$

$$= 27 - 12 - 9 = 6 > 0$$

$$F(2) \cdot F(3) = 0$$

The root lies b/w 2 and 3

1st Approximation $x_1 = \frac{a_1 + b_1}{2} = \frac{2+3}{2} = \frac{5}{2} = 2.5$

$F(2.5) = (2.5)^3 - 4(2.5) - 9$

$= 15.625 - 10 - 9 = -3.375 < 0$

$F(2.5) \neq 0 \quad F(2.5) < 0$

$F(3) > 0 \Rightarrow F(2.5)\, F(3)$

Now the root lies between 2.5, 3

$a_2 = 2.5 \quad b_2 = 3$

2nd Approximation $x_2 = \frac{2.5+3}{2} = \frac{5.5}{2} = 2.75$

$F(2.75) = (2.75)^3 - 4(2.75) - 9$

$= 20.7969 - 11 - 9$

$= 0.7969 > 0$

$F(2.75) > 0, \; F(2.5) < 0$

$\therefore$ The root lies between $(2.5, 2.75)$

$a_3 = 2.5 \quad b_3 = 2.75$

3rd Approximation $x_3 = \frac{2.5+2.75}{2} = \frac{5.25}{2} = 2.625$

$F(2.625) = (2.625)^3 - 4(2.625) - 9$

$= 18.08789 - 10.5 - 9$

$= 18.08789 - 19.5$

$= -1.4121 < 0$

$F(2.625) < 0, \; F(2.75) > 0$

$\therefore$ The root lies between 2.625, 2.75

$a_4 = 2.625$ and $b_4 = 2.75$

4^{th} Approximation.

$$x_4 = \frac{2.625 + 2.75}{2}$$

$$= \frac{5.375}{2} = 2.6875$$

$$F(2.6875) = (2.6875)^3 - 4(2.6875) - 9$$

$$= 19.4109 - 10.75 - 9$$

$$= 19.4109 - 19.75$$

$$= -0.3391$$

Result:-

Table:-

n	a_n	b_n	x_n	$F(x_n)$
1	2	3	2.5	-3.375
2	2.5	3	2.75	0.797
3	2.5	2.75	2.625	-1.4121
4	2.625	2.75	2.6875	-0.3391

Find a real root of the Equation $f(x) = x^3 - 2x - 5 = 0$ by the method of the False position upto 3 decimal places.

Solution :-

Aim :- A real root of the equation $f(x) = x^3 - 2x - 5 = 0$ by Method of the False position up to 3 decimal places

Formula :-

$$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

procedure :-

Given $f(x) = x^3 - 2x - 5 = 0$

$$f(1) = 1^3 - 2(1) - 5 = -6 < 0$$

$$f(2) = 2^3 - 2(2) - 5 = -1 < 0$$

$$f(3) = 3^3 - 2(3) - 5 = 16 > 0$$

$$f(2) \cdot f(3) = -1 \times 16 < 0$$

The root lies b/w 2 and 3

$$a_1 = 2, \; b_1 = 3$$

1st Approximation of the root is.

$$x_1 = \frac{a_1 f(b_1) - b_1 f(a_1)}{f(b_1) - f(a_1)}$$

$$= \frac{2 \times 16 - 3(-1)}{16 + 1} = \frac{32 + 3}{17} = \frac{35}{17} = 2.0588$$

$f(x_1) = (2.0588)^3 - 2(2.0588) - 5$

$= -0.39105 < 0$

$f(3) \geq 0$

$\therefore$ The root lies between 2.0588 and 3

$a_2 = 2.0588 \quad b_2 = 3$

2nd Approximation of the root is

$$x_2 = \frac{a_2 f(b_2) - b_2 f(a_2)}{f(b_2) - f(a_2)}$$

$$= \frac{2.0588(16) - 3(0.39105)}{16 + 0.39105}$$

$$= \frac{32.9408 + 1.1732}{16.39105} = \frac{34.114}{16.39105}$$

$$= 2.08125$$

$f(x_2) = f(2.08125) = (2.08125)^3 - 2(2.08125) - 5$

$= -0.14735 < 0$

$= -0.1468 < 0$

The root lies b/w $2.08\cancel{1245}$ & 3

$a_3 = 2.08125 \quad b_3 = 3$

3rd Approximation of the root is

$$x_3 = \frac{a_3 f(b_3) - b_3 f(a_3)}{f(b_3) - f(a_3)}$$

$$= \frac{2.08125(16) + 3(0.1468)}{16 + 0.1468}$$

$$= 2.0897$$

$$f(2.0928) = (2.0928)^3 - 2(2.0928) - 5$$

$$= -0.0195 < 0$$

The root lies b/w 2.0928 & 3

$a_5 = 2.0928$, $b_5 = 3$

5^{th} Approximation $$x_5 = \frac{2.0928(16) + 3(0.0195)}{16 + 0.0195}$$

$$= 2.0939.$$

$$f(2.0939) = (2.0939)^3 - 2(2.0939) - 5$$

$$= -0.0072 < 0$$

The root lies b/w 2.0939 & 3

$a_6 = 2.0939$, $b_6 = 2.0939$

6^{th} Approximation $$x_6 = \frac{2.0939(16) + 3(0.0072)}{16 + 0.0072}$$

$$= 2.0943$$

$$f(2.0943) = (2.0943)^3 - 2(2.0943) - 5$$

$$= -0.0028$$

$$f(3) = 16 > 0$$

The root lies b/w 2.0943 & 3

$a_7 = 2.0943$, $b_7 = 3$

7^{th} Approximation $$x_7 = \frac{a_7 f(b_7) - b_7 (f a_7)}{f(b_7) - f(a_7)}$$

$$= \frac{2.0943(16) + 3(0.0028)}{16 + 0.0028}$$

$$= \frac{33.5088+0.0084}{16.0028}$$

$$= \frac{33.5172}{16.0028}$$

$$= 2.0945$$

∴ Hence the root is 2.0945 correct to 3 decimal places.

Result :- The regula root of the equation is 2.0945,

correct to 3 decimal places,

UNIT-3

INTERPOLATION-I

INTERPOLATION-I

Introduction:-

If $f(x)$ is a continuous single valued function then the values of $f(x)$ corresponding to certain given values of x. say $x_0, x_1, x_2 ---- x_n$ can easily (be computed and value) be computed and tabulatet the converse one. given the set of tabularvalues $[x_0, y_0] [x_1, y_1] [x_2, y_2] ----- [x_n, y_n]$ satisfiying the relation $y = f(x)$ when the explicit nature of $f(x)$ is not known. It is required to find a simple function say $\phi(x)$, such that $f(x)$ and $\phi(x)$ agree at the set of tabulated points such a porces is called interpolation. It $\phi(x)$ is a polynomial then the process is called polynomial interpolation $\phi(x)$ is called the interpolatiang polynomial

Statement of the problem

To prove that (i) $\Delta = E-1$

(ii) $\nabla = \cancel{1} - E^{-1}$

(iii) $E = e^{hD}$

Solution :-

Aim :- $\Delta = E-1$, (ii) $\nabla = 1-E^{-1}$ (iii) $E = e^{hD}$

Formula :- $\Delta f(x) = f(x+h) - f(x)$

procedure:-

(i) $\Delta f(x) = f(x+h) - f(x)$

$= Ef(x) - f(x)$

$= (E-1) f(x)$

$\Delta f(x) = (E-1) f(x)$

$\Delta = E-1$

(ii) $\nabla f(x) = f(x) - f(x-h)$

$= (1-E^{-1}) f(x)$

$= (1-E^{-1}) f(x)$

$\Delta = (1-E^{-1})$

(iii) $Ef(x) = f(x+h)$

$$= f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{3}f'''(x) + \cdots$$

$$= f(x) + hDf(x) + \frac{h^2}{2}D^2f(x) + \frac{h^3}{3}D^3f(x) + \cdots$$

$$= \left(1 + hD + \frac{h^2}{2}D^2 + \frac{h^3}{3}D^3 + \cdots\right)f(x)$$

$$Ef(x) = e^{hD}f(x)$$

$$E = e^{hD}$$

Result:-

(i) $\Delta = E - 1$

(ii) $\nabla = 1 - E^{-1}$

(iii) $E = e^{hD}$

statement of the problem

Given $u_0=3$, $u_1=12$, $u_2=81$, $u_3=200$, $u_4=100$, $u_5=8$. Find $\Delta^5 y_0$.

Solution :-

Aim :- Given $u_0=3$, $u_1=12$, $u_2=81$, $u_3=200$, $u_4=100$

$u_5=8$. Find $\Delta^5 y_0$

Formula :- $\Delta^5 y_0=(E-1)^5 y_0=(5C_0E^5-5C_1E^4+5C_2E^3+5C_3E^2+5C_4E-5C_5)$ u_0.

procedure :-

Given $u_0=3$, $u_1=12$, $u_2=81$, $u_3=200$, $u_4=100$

$\Delta^5 y_0=(E-1)^5 y_0=[5C_0E^5-5C_1E^4+5C_2E^3-5C_3E^2+5C_4E+5C_5]\,u_0$

$=[E^5-5E^4+10E^3-10E^2+5E-1]\,u_0$

$=E^5u_5-5E^4u_4+10E^3u_3-10E^2u_2+5Eu_1-u_0$

$=\cancel{E^5}u_5-5\cancel{E^4}u_4+10u_3-10u_2+5u_1-u_0$

$=8-5(100)+10(200)-10(81)+5(12)-3$

$=755$

Result :- $\Delta^5 y_0=755$

UNIT-IV

INTERPOLATION-II

INTERPOLATION - II

Interpolation With Equal Intervals.

Introduction :-

Let $f(x_0), f(x_1) \cdots\cdots f(x_n)$ be the set of functional values for the function $y=f(x)$ at $x=x_0, x_1, x_2 \cdots\cdots x_n$ and we have to find some intermediate values of the function from the given set of values for example $\sin\theta$ is known for $\theta = 30°, 45°, 60°, 75°, 90°$ and we have to compute the value of $\sin 70°$ from the set of values known

The process of computing intermediate value is called the interpolation according to their is as the out of the reading between the lines of the table".

Statement of the problem

NEWTON'S FORMULAS OF INTERPOLATION

Statement :- Newton-Gregory formula for forward interpolation with equal internals, If $y=f(x)$ is a function which assumes the values $y_0, y_1, y_2, \dots\dots y_n$ for $(n+1)$ Equidistant values $x_0, x_1, x_2, \dots\dots, x_n$ where $x_0=a$, $x_1=a+h$, $x_2=a+2h \dots\dots x_n=a+nh$ of the argument x

then, $y_u = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \dots\dots + \frac{u(u-1)\dots(u-n+1)}{n!}\Delta^n y_0$

where $u = \frac{x-x_0}{h}$.

Solution :-

Aim :- $y_u = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \dots\dots + \frac{u(u-1)\dots(u-n+1)}{n!}\Delta^n y_0$

Formula :-

$$u = \frac{x-x_0}{h} \Rightarrow x-x_0 = uh.$$

$$x = x_0+uh.$$

procedure :-

Here we may assumed that $f(x)$ is a polynomial of degree n Now $f(x)$ may be written as

$$f(x) = A_0 + A_1(x-x_0) + A_2(x-x_0)(x-x_1) + A_3(x-x_0)(x-x_1)(x-x_2) + \dots\dots + A_n(x-x_0)(x-x_1)\dots\dots(x-x_{n-1}) \quad (1)$$

where $A_0, A_1, A_2, \ldots A_n$ are constants.

putting $x = x_0$ in eqn ①, we get

$$f(x_0) = A_0 \Rightarrow A_0 = y_0$$

putting $x = x_1$ in eqn ① we get

$$f(x_1) = A_0 + A_1(x_1 - x_0)$$

$$y_1 = y_0 + A_1 h$$

$$A_1 = \frac{y_1 - y_0}{h}$$

$$A_1 = \frac{\Delta y_0}{h}$$

putting $x = x_2$ in eqn ① we get

$$f(x_2) = A_0 + A_1(x_2 - x_0) + A_2(x_2 - x_0)(x_2 - x_1)$$

$$y_2 = y_0 + \frac{\Delta y_0}{h} \times 2h + A_2(2h \times h)$$

$$= y_0 + 2\Delta y_0 + A_2(2h^2)$$

$$A_2(2h^2) = y_2 - y_0 - 2\Delta y_0$$

$$(2h^2 \times A_2) = y_2 - y_0 - 2(y_1 - y_0)$$

$$= y_2 - 2y_1 + y_0$$

$A_2 = \frac{\Delta^2 y_0}{2!\,h^2}$ proceeding in this way we get

$$A_3 = \frac{\Delta^3 y_0}{3!\,h^3}, \quad A_4 = \frac{\Delta^4 y_0}{4!\,h^4} \ldots \ldots A_n = \frac{\Delta^n y_0}{n!\,h^n}$$

Substituting $A_0, A_1, A_2, \ldots A_n$ values in ① we get

$$f(x) = y_0 + \frac{\Delta y_0}{h}(x-x_0) + \frac{\Delta^2 y_0}{2!\,h^2}(x-x_0)(x-x_1) + \cdots + \frac{\Delta^n y_0}{n!\,h^n}(x-x_0)(x-x_1)\cdots(x-x_{n-1}) \quad \text{(2)}$$

putting $u = \frac{x-x_0}{h}$ so that $x = x_0 + hu$ we get

$$f(x_0+hu) = y_0 + \frac{\Delta y_0}{h}(hu) + \frac{\Delta^2 y_0}{2!\,h^2}(hu)(hu-h) + \frac{\Delta^3 y_0}{3!\,h^3}(hu)(hu-h)(hu-2h) + \cdots + \frac{\Delta^n y_0}{n!\,y^n}(hu)(hu-h)(hu-2h)\cdots\left[hu(n-1)\atop n\right]$$

$$= y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \cdots + \frac{u(u-1)(u-2)\cdots(u-n+1)}{n!}\Delta^n y_0$$

$$\therefore\ y_u = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \cdots + \frac{u(u-1)(u-2)\cdots(u-n+1)}{n!}\Delta^n y_0$$

Result:-

$$y_u = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \cdots + \frac{u(u-1)(u-2)\cdots(u-n+1)}{n!}\Delta^n y_0$$

Gauss Forward Formula

INTRODUCTION:-

Interpolation refers of the process of creating new data points given within the given set of data. The below code computer desired data point within the given range of discreate data sets using the formula given by Gauss and this method known as Gauss's forward method.

If $y=f(x)$ is a function, which takes the values ... $\dots y_{-2}, y_{-1}, y_0, y_1, y_2 - - - -$ corresponding to the values of $x = - - - - - - - x_0-2h, x_0-h, x_0, x_0+h, x_0+2h + - - -$ them

$$y_u = y_0 + \Delta u y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{(u+1)u(u-1)}{3!}\Delta^3 y_{-1} + \frac{(u+1)u(u-1)(u-2)}{4!}\Delta^4 y_{-2} + - - - - - -$$

where $u = \frac{x-x_0}{h}.$

Statement of the problem :-

using Gauss Forward formula find u_{32} from the given data $u_{20} = 14.035$, $u_{25} = 13.674$, $u_{30} = 13.257$, $u_{35} = 12.734$, $u_{40} = 12.089$, $u_{45} = 11.309$

Solution :-

Aim :- Gauss forward formula find u_{32} from the given data. $u_{20} = 14.035$, $u_{25} = 13.674$, $u_{30} = 13.257$, $u_{35} = 12.734$, $u_{40} = 12.089$, $u_{45} = 11.309$

Formula :- By Gauss forward interpolation formula

$$y_u = y_0 + u\Delta y_0 + \frac{u(u-1)}{2}\Delta^2 y_{-1} + \frac{(u+1)u(u-1)}{6}\Delta^3 y_{-1} + \frac{(u+1)u(u-1)(u-2)}{24}\Delta^4 y_{-2} + \frac{(u+2)u(u-1)(u-2)}{120}\Delta^5 y_{-2}$$

procedure :-

Let us take the origin at $x=30$ and $h=5$ as the unit.

To find the value of y at $x=32$,

$$u = \frac{32-30}{5}$$

$$= 0.4.$$

x	u	y_u	Δy_u	$\Delta^2 y_u$	$\Delta^3 y_u$	$\Delta^4 y_u$	$\Delta^5 y_u$
20	-2	14.035					
			-0.361				
25	-1	13.674		-0.056			
			-0.417		-0.050		
30	0	13.257		-0.106		-0.034	
			-0.523		-0.016		-0.031
35	1	12.734		-0.122		0.003	
			-0.645		-0.013		
40	2	12.089		-0.135			
			-0.780				
45	3	11.309					

By Gauss forward interpolation formula

$$y_u = y_0 + u\Delta y_0 + \frac{u(u-1)}{2}\Delta^2 y_{-1} + \frac{(u+1)u(u-1)}{6}\Delta^3 y_{-1} + \frac{(u+1)u(u-1)(u-2)}{24}\Delta^4 y_{-2}$$

$$+ \frac{(u+2)u(u-1)(u-2)}{120}\Delta^5 y_{-2}$$

$$\therefore y_{0.4} = 13.257 + 0.4(-0.523) + \frac{0.4(0.4-1)}{2}(-0.106) + \frac{(0.4+1)(0.4)(0.4-1)}{6}$$

$$(-0.016) + \frac{(0.4+1)0.4(0.4-1)(0.4-2)}{24}(0.003)$$

$$+ \frac{(0.4+2)(0.4+1)(0.4)(0.4-1)(0.4-2)}{120}(-0.031)$$

$$= 13.257 - 0.2092 + 0.01272 + 0.000896 + 0.0000672$$

$$-0.000983312$$

$$= 13.257 - 0.2092 + 0.01272 + 0.000896 + 0.0000672 - 0.000833312$$

$$= 13.06115$$

$$= 13.061.$$

Result:- 13.061

UNIT-V

INTERPOLATION - III

NEWTON'S DIVIDED DIFFERNCE FORMULA

INTRODUCTION:-

This formula is called Newton's divided difference formula once we have the divided difference of the function f relative to the tabular points then we can use the above formula to computer f(x) at any non tabular point

$$f(x) = F(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2)$$

Statement of the problem

By means of Newton's divided difference formula, Find the value f(8) and f(15) from the following table:

x:	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

Solution :-

Aim :- A Newton's divided difference formula. Find the value f(8) and f(15) from

Formula :-

$$f(x) = f(x_0) + (x-x_0) f(x_0,x_1) + (x-x_0)(x-x_1) f(x_0,x_1,x_2) + (x-x_0)(x-x_1)(x-x_2) f(x_0,x_1,x_2,x_3) \quad \text{—— (1)}.$$

procedure :-

Here divided difference table is given below.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	~~$\Delta^5 f(x)$~~
4	48					
		$\frac{100-48}{5-4}=52$				
			$\frac{97-52}{7-4}=15$			
				$\frac{21-15}{10-4}=1$		
5	100				0	
		$\frac{294-100}{7-5}=97$				
			$\frac{202-97}{10-5}=21$			
				$\frac{27-21}{11-5}=1$		
7	294					
		$\frac{900-294}{10-7}=202$			0	
			$\frac{310-202}{11-7}=27$			
10	900			$\frac{33-27}{13-7}=1$		
		$\frac{1210-900}{11-10}=310$				
			$\frac{409-310}{13-10}=33$			
11	1210					
		$\frac{2028-1210}{13-11}=409$				
13	2028					

Now using Newton's divided difference formula, we get

$$f(x) = f(x_0) + (x-x_0)f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0,x_1,x_2) + (x-x_0)(x-x_1)(x-x_2)f(x_0,x_1,x_2,x_3) \quad \text{——} \; (1).$$

$$f(8) = f(4) + (8-4)\times 52 + (8-48)(8-5)15 + (8-4)(8-5)(8-7)\times 1$$

$$= 48 + 208 + 180 + 12$$

$$= 448$$

and

$$f(15) = 48 + (15-4)\times 52 + (15-4)(15-5)(15) + (15-4)(15-5)(15-7)1$$

$$= 48 + 572 + 1650 + 880$$

$$\therefore f(15) = 3150.$$

Result:- $f(8) = 448$

and $f(15) = 3150$

Statement of the problem

Lagrange's interpolation formula

Statement:- Let $y=f(x)$ be a function, which takes the values $f(x_0), f(x_1), \ldots f(x_n)$ corresponding to the values of $x=x_0, x_1, x_2, \ldots x_n$, not necessarily equally spaced. Then.

$$f(x)=\frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)}f(x_0)+\frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)}f(x_1)+\ldots+\frac{(x-x_0)(x-x_1)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots(x_n-x_{n-1})}f(x_n)$$

Solution :-

Aim :- Lagrang's interpolation formula.

Formula:-

$$f(x)=\frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)}f(x_0)+\frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)}f(x_1)+\ldots+\frac{(x-x_0)(x-x_1)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots(x_n-x_{n-1})}f(x_n).$$

Procedure:-

Let $y=f(x)$ be a function which can assume the values $f(x_0), f(x_1), \ldots f(x_n)$ corresponding to the values of the values of the arguments $x_0, x_1, \ldots, x_n$ respectively

Now, since there are $(n+1)$ be a function which can pair of values of x and y, therefore, we can represent $f(x)$ by a polynomial in x of degree n.

Let this polynomial be of the form

$$f(x) \approx L_n(x) = A_0(x-x_1)(x-x_2)\text{----}(x-x_n) + A_1(x-x_0)(x-x_2)\text{---}(x-x_n)$$
$$+ A_2(x-x_0)(x-x_1)(x-x_3)\text{----}(x-x_n) + \text{----} + A_n(x-x_0)$$
$$(x-x_1)(x-x_2)\text{----}(x-x_{n-1}) \quad \text{—— (1).}$$

where $A_0, A_1, A_2, \text{----} A_n$ are constants and can be determined by putting $f(x) = f(x_0)$ at $x = x_0$, $f(x) = f(x_1)$ at $x = x_1$, ---- etc.

Now put $x = x_0$ and $f(x) = f(x_0)$ in (1), we get $f(x_0) = A_0(x_0-x_1)$
$(x_0-x_2)\text{---}(x_0-x_n)$

$$\Rightarrow A_0 = \frac{f(x_0)}{(x_0-x_1)(x_0-x_2)\text{---}(x_0-x_n)}$$

Similarly, by putting $f(x) = f(x_1)$ and $x = x_1$ in (1) we get

$$f(x_1) = A_1(x_1-x_0)(x_1-x_2)\text{---}(x_1-x_n)$$

$$\Rightarrow A_1 = \frac{f(x_1)}{(x_1-x_0)(x_1-x_2)\text{----}(x_1-x_n)}$$

Proceeding in the same way we get the values of $A_2, A_3,$ $A_4, \text{----} A_n$, as.

$$A_i = \frac{f(x_i)}{(x_i-x_0)(x_i-x_1)\cdots(x_i-x_{i-1})(x_i-x_{i+1})\cdots(x_i-x_n)}$$ for $i = 2, 3 \ldots n$.

By putting all these values of $A_0, A_1, \ldots A_n$ in (1) we get

$$f(x) \approx L_n(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} f(x_0) +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\cdots(x_1-x_n)} f(x_1) + \cdots$$

$$+ \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} f(x_n)$$

Which is the required Lagrange's interpolation formula.

Result:- the required Lagrange's interpolation formula is proved.

ADVANCE NUMERICAL ANALICIS

UNIT-1

ADVANCED NUMERICAL ANALYSIS

Curve Fitting

Curve Fitting

Introduction:-

In this chapter we are concerned with the problem of fitting an Equation or a curve to data involving paired values the exact Mathematical relationship between two variable is given by simple algebraic expression called curve fitting are relatively limited in type the straight line is the simplest and one of the most important curve used.

Statement of the problem.

Find the least square line for the data points $(-1,10)$, $(0,9)$, $(1,7)$, $(2,5)$, $(3,4)$, $(4,3)$, $(5,0)$ and $(6,-1)$

Solution :-

Aim :- The least square line for the data points $(-1,10)$ $(0,9)$, $(1,7)$, $(2,5)$, $(3,4)$ $(4,3)$, $(5,0)$ and $(6,-1)$

Formula :-

The normal equation for straight line are

$$na + b\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i$$

$$a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i y_i$$

procedure :-

x_i	y_i	x_i^2	$x_i y_i$
-1	10	1	-10
0	9	0	0
1	7	1	7
2	5	4	10
3	4	9	12
4	3	16	12
5	0	25	0
6	-1	36	-6
$\Sigma x_i = 20$	$\Sigma y_i = 37$	$\Sigma x_i^2 = 92$	$\Sigma x_i y_i = 25$

The normal equation for straight are

$$na + b\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i \quad \text{—— (1)}$$

$$a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i y_i \quad \text{—— (2)}$$

here $n = 8$

Eqn (1). $8a + b(20) = 37$

$$8a + 20b = 37 \quad \text{—— (3).}$$

Eqn (2) $20a + b(92) = 25$

$$20a + 92b = 25 \quad \text{—— (4).}$$

Eqn (3) x3 - (4) eqn

$$24a + 60b = 111$$

$$20a + 92b = 25$$

$$4a - 32b = 86 \quad \text{—— (5)}$$

Eqn (4) - Eqn (5) x5

$$20a + 92b = 25$$

$$20a + 160b = 430$$

$$- \quad + \quad -$$

$$252b = -405$$

$$b = -\frac{405}{252} \qquad \boxed{b = -1.6071}$$

from eqn (3).

$$8a + 20(-1.6) = 37$$

$$8a - 32 = 37$$

$$8a = 37 + 32$$

$8a = 69$

$a = \frac{69}{8}$

$\boxed{a = 8.625}$

$y = a+b$

$= (8.625)x + (-1.6)$

$= 8.625x - 1.6.$

Result :-

The least saure line is $y = 8.625x - 1.6071$,

Statement of the problem

Find the least squre power function of the form $y=ax^b$ for the data

x_i	1	2	3	4
y_i	3	12	21	35

Solution :-

Aim :- The least squre power function of the form

$$y=ax^b$$

Formula:- The normal equation are

$$nA + B\sum_{i=1}^{n} X_i = \sum_{i=1}^{n} Y_i \quad \text{—— (1)}$$

$$A\sum_{i=1}^{n} X_i + B\sum_{i=1}^{n} X_i^2 = \sum_{i=1}^{n} X_i Y_i \quad \text{—— (2).}$$

procedure:-

x_i	y_i	$X_i=\log x_i$	$Y_i=\log y_i$	X_i^2	X_iY_i
1	3	0	1.0986	0	0
2	12	0.6931	2.4849	0.4804	1.1937
3	21	1.0986	3.0445	1.2069	3.6744
4	35	1.3863	3.5553	1.9218	6.8326
$\Sigma x_i=10$	$\Sigma y_i=71$	$\Sigma X_i=3.1780$	$\Sigma Y_i=10.1833$	$\Sigma X_i^2=3.6091$	$\Sigma X_iY_i=11.7007$

The normal equation are

hence $n=4$

$$nA + B\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i \quad \text{——} (1)$$

$$A\sum_{i=1}^{n} x_i + B\sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i y_i \quad \text{——} (2)$$

$$4A + (3.1780)B = 10.1833$$

$$(3.1780)A + (3.6091)B = 11.7007$$

By solving we get

$$A = -0.0997, \quad B = 3.3298$$

$\therefore$ The least saure function is $Y = -0.0997 + 3.3298X$

$$\Rightarrow \log y = -0.0997 + 3.3298 \log x$$

$$\Rightarrow y = 0.9051 e^{3.3298x}$$

Result :- $y = \cancel{0.99}\ 0.9051 e^{3.3298x}$.

UNIT-II

Numerical Differentiation

Statement of the problem :-

Derivaties using stirriling's Formula

using the given table, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.2$

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

Solution :-

Aim :- The given table, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.2$

Formula :-

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0 - \frac{1}{6}\Delta^6 y_0 + \cdots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \cdots\right]$$

procedure :- Difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1.0	2.7183						
		0.6018					
1.2	3.3201		0.1333				
		0.7351		0.0294			
1.4	4.0552		0.1627		0.0067		
		0.8978		0.0361		0.0013	
1.6	4.9530		0.1988		0.0080		0.0001
		1.0966		0.0441		0.0014	
1.8	6.0496		0.2429		0.0094		
		1.3395		0.0535			
2.0	7.3891		0.2964				
		1.6359					
2.2	9.0250						

We have $\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0 - \frac{1}{6}\Delta^6 y_0\right]$

Here $x_0 = 1.2$, $h = 0.2$ $\Delta y_0 = 0.7351$, $\Delta^2 y_0 = 0.1627$ etc--- we get

$$\frac{dy}{dx} = \frac{1}{0.2}\left[0.7351 - \frac{1}{2}(0.1627) + \frac{1}{3}(0.0361) - \frac{1}{4}(0.0080) + \frac{1}{5}(0.0014)\right]$$

$$= \frac{1}{0.2}\left[0.7351 - 0.08135 + 0.0120 - 0.002 + 0.00028\right]$$

$$= 3.32015$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0\right]$$

$$= \frac{1}{0.04}\left[0.1627 - 0.0361 + \frac{11}{12}(0.0080) - \frac{5}{6}(0.0014)\right]$$

$$= \frac{1}{0.04}\left[0.1627 - 0.0361 + 0.0073 - 0.0012\right]$$

$$= 3.3175.$$

Result :-

$$\therefore \frac{dy}{dx} = 3.32015$$

and

$$\frac{d^2y}{dx^2} = 3.3175$$

Statement of the problem

From the following table, find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at

$x = 203.$ $x = 2.03$

x	1.96	1.98	2.00	2.02	2.04
y	0.7825	0.7739	0.7651	0.7563	0.7473

Solution :-

Aim :- The following table, find the values of $\frac{dy}{dx}$ and

$\frac{d^2y}{dx^2}$ at $x = 2.03$

Formula :- Newton's backward formula

$$y(x) = y_n + u\nabla y_n + \frac{u(u+1)}{2}\nabla^2 y_n + \frac{u(u-1)(u-2)}{6}\nabla^3 y_n + \text{------}$$

Procedure :- Difference table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1.96	0.7825				
		-0.0086			
1.98	0.7739		-0.0002		
		-0.0088		0.0002	
2.00	0.7651		-0		-0.0004
		-0.0088		-0.0002	
2.02	0.7563		-0.0002		
		-0.0090			
2.04	0.7473				

Here $x_n = 2.04$, $h = 0.02$, $u = \frac{x - x_n}{h} \Rightarrow \frac{du}{dx} = \frac{1}{h}$ at $x = 2.03$

$$u = \frac{2.03 - 2.04}{0.02} = -\frac{0.01}{0.02} = -\frac{1}{2}$$

Then by Newton's backward formula, we have

$$y(x) = y_n + u\nabla y_n + \frac{u(u+1)}{2}\nabla^2 y_n + \frac{u(u+1)(u+2)}{6}\nabla^3 y_n + \frac{u(u+1)(u+2)(u+3)}{24}\nabla^4 y_n + \cdots \quad \to (1)$$

Differentiating w.r.to. x, we get

$$y'(x) = \frac{1}{h}\left[\nabla y_n + \frac{2u+1}{2}\nabla^2 y_n + \frac{3u^2+6u+2}{6}\nabla^3 y_n + \frac{4u^3+18u^2+22u+6}{24}\nabla^4 y_n + \cdots\right] \quad \to (2)$$

$$y'(2.03) = \frac{1}{0.02}\left[-0.0090 + 0.0001 + \frac{3(-1/2)^2+6(-1/2)+2}{6}(-0.0002) + \frac{4(-1/2)^3+18(-1/2)^2+22(-1/2)+6}{24}(-0.0004)\right]$$

$$= \frac{1}{0.02}\left[-0.0090 + 0.000008 + 0.000017\right]$$

$$\therefore \frac{dy}{dx} = -0.44875$$

Again differentiating equation (2) w.r.t x,

$$y''(x) = \frac{1}{h^2}\left[\nabla^2 y_n + \frac{6u+6}{6}\nabla^3 y_n + \frac{12u^2+36u+22}{24}\nabla^4 y_n + \cdots\right]$$

$$y''(2.03) = \frac{1}{(0.02)^2}\left[-0.0002 + \left(-\frac{1}{2}+1\right)(-0.0002) + \frac{12(-1/2)^2+36(-1/2)+22}{24}(-0.0004)\right]$$

$$= \frac{1}{0.0004}\left[-0.0002 - 0.0001 - 0.00012\right]$$

$$\therefore \frac{d^2y}{dx^2} = -1.05$$

Result :- $\therefore \dfrac{dy}{dx} = -0.44875$

and

$$\frac{d^2y}{dx^2} = -1.05$$

UNIT-III

Numerical Integration

Statement of the problem :-

Find the maximum value of y, by using data given below

x	0	1	2	3	4
y	0	0.25	0	2.25	16

Solution :-

Aim :- The maximum value of y, by using data given below table

Formula :-

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{2u-1}{2}\Delta^2 y_0 + \frac{3u^2-6u+2}{6}\Delta^3 y_0 + \frac{4u^3-18u^2+22u-6}{24}\Delta^4 y_0\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \frac{6u^2-18u+11}{12}\Delta^4 y_0\right]$$

Procedure :-

The forward difference table is.

x	y	Δ	Δ^2	Δ^3	Δ^4
0	0				
		0.25			
1	0.25		-0.5		
		-0.25		3	
2	0		2.5		6
		2.25		9	
3	2.25		11.5		
		13.75			
4	16				

We have $x_0 = 0$, $h = 1$

Differentiating Newton - Gregory forward interpolation formula, we get

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{2u-1}{2!}\Delta^2 y_0 + \frac{3u^2-6u+2}{3!}\Delta^3 y_0 + \frac{4u^3-18u^2+22u-6}{4!}\Delta^4 y_0 + \cdots\right]$$

$$= \frac{1}{1}\left[0.25 + \frac{2u-1}{2}(-0.5) + \frac{3u^2-6u+2}{6}(3) + \frac{4u^3-18u^2+22u-6}{24}(6)\right]$$

$$= \frac{1}{4} - \frac{2u-1}{4} + \frac{3u^2-6u+2}{2} + \frac{4u^3-18u^2+22u-6}{4}$$

$$= \frac{4u^3-12u^2-8u}{4}$$

$$= u^3-3u^2-2u$$

$$\frac{dy}{dx} = 0 \Rightarrow u^3-3u^2-2u = 0$$

$$u = 0 \text{ or } 1, 2$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \frac{6u^2-18u+11}{12}\Delta^4 y_0\right]$$

$$= \frac{1}{(1)^2}\left[-0.5 + (u-1)(3) + \frac{6u^2-18u+11}{12}(6)\right]$$

$$= -0.5 + 3u - 3 + 3u^2 - 9u + \frac{11}{12}$$

$$= 3u^2-6u+2$$

$$\frac{d^2y}{dx^2} = 3-6+2 = -1 < 0$$

$\therefore$ y has maximum when $u=1$, maximum value $= 0.25$

Result :-

$\therefore$ y has maximum when $u=1$, maximum value $= 0.25$

Statement of the problem

Calculate an approximate value of $\int_0^{\pi/2} \sin x\,dx$ simpson's $\frac{1}{3}$ rule, using 11 ordinates.

Solution :-

Aim :- An approximate value of $\int_0^{\pi/2} \sin x\,dx$ simpson's $\frac{1}{3}$ rule, using 11 ordinates.

Formula :- By simpson's $\frac{1}{3}$ rule

$$\int_0^{\pi/2} \sin x\,dx = \frac{h}{3}\left[(y_0+y_{10})+4(y_1+y_3+y_5+y_7+y_9)+2(y_2+y_4+y_6+y_8)\right]$$

Procedure:-

Divide the interval $[0,\pi/2]$ into 10 equal subintervals of which

$$h = \frac{\pi/2-0}{10} = \frac{\pi}{20}$$

The values of $y=f(x)=\sin x$ are given at each point of sub division as follows

x	0	$\pi/20$	$2\pi/20$	$3\pi/20$	$4\pi/20$	$5\pi/20$
$y=\sin x$	0	0.1564	0.3090	0.4540	0.5878	0.7071

x	$6\pi/20$	$7\pi/20$	$8\pi/20$	$9\pi/20$	$\pi/2$
$y=\sin x$	0.8090	0.8910	0.9510	0.9877	1

Here $y_0=0$, $y_1=0.1564$, $y_2=0.3090$, $y_3=0.4540$, $y_4=0.5878$

$y_5=0.7071$, $y_6=0.8090$, $y_7=0.8910$, $y_8=0.9510$ $y_9=0.9877$

$y_{10}=1$.

By simpson's $\frac{1}{3}$ rule,

$$\int_0^{\pi/2} \sin x\,dx = \frac{h}{3}\left[(y_0+y_{10})+4(y_1+y_3+y_5+y_7+y_9)+2(y_2+y_4+y_6+y_8)\right]$$

$$= \frac{\pi}{20}\left[(0+1)+4(0.1564+0.4540+0.7071+0.8910+0.9877)+\right.$$

$$\left.2(0.3090+0.5878+0.8090+0.9510)\right]$$

$$= \frac{11}{70}\left[1+4(3.1962)+2(2.6568)\right]$$

$$= \frac{11}{70}\left[1+12.7848+5.3136\right]$$

$$= \frac{11}{70}\left[18.0984\right]$$

$$= 2.8440.$$

Result :-

$$\int_0^{\pi/2} \sin x\,dx = 2.8440.$$

Statement of the problem

Find the value of the integral $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule. Hence obtain the approximate value of π in each case

Solution :-

Aim :- The value of the integral $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule.

Formula :-

By Simpson's $\frac{1}{3}$ rule.

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{3}\left[y_0+4(y_1+y_3+y_5)+2(y_2+y_4)+y_n\right]$$

By Simpson's $\frac{3}{8}$ rule

$$\int_0^1 \frac{dx}{1+x^2} = \frac{3h}{8}\left[y_0+3(y_1+y_2+y_4+y_5)+2y_3+y_6\right]$$

Procedure :- Divide the interval $[0,1]$ into six equal subinterval of width $h=\frac{1}{6}$. Then the values of $y=f(x)=\frac{1}{1+x^2}$ at each points of sub-divisions are given below.

x	0	1/6	2/6	3/6	4/6	5/6	1
$y=\frac{1}{1+x^2}$	1.0000	0.9729	0.9000	0.8000	0.6923	0.5901	0.5000

Here $y_0=1.0000$, $y_1=0.9729$, $y_2=0.9000$, $y_3=0.8000$, $y_4=0.6923$, $y_5=0.5901$ and $y_6=0.5000$

By Simpson's $\frac{1}{3}$ rule $\int_0^1 \frac{dx}{1+x^2} = \frac{h}{3}\left[y_0+4(y_1+y_3+y_5)+2(y_2+y_4)+y_6\right]$

$$= \frac{1}{18}\left[1.0000 + 4(0.9729 + 0.8000 + 0.5901) + 2(0.9000 + 0.6923) + 0.5000\right]$$

$$= \frac{1}{18}\left[1.5 + 4(2.363) + 2(1.5923)\right]$$

$$= \frac{1}{18}\left[1.5 + 9.452 + 3.1846\right]$$

$$= \frac{1}{18}\left[14.1366\right]$$

$$= 0.785366$$

By simpson's $\frac{3}{8}$ rule :- $\int_0^1 \frac{dx}{1+x^2} = \frac{3h}{8}\left[y_0 + 3(y_1+y_2+y_4+y_5) + 2y_3 + y_6\right]$

$$= \frac{3\times\frac{1}{6}}{8}\left[1.0000 + 3(0.9729 + 0.9000 + 0.6923 + 0.5901) + 2(0.8000) + 0.5000\right]$$

$$= \frac{1}{16}\left[1.5 + 3(3.1553) + 1.6\right]$$

$$= \frac{1}{16}\left[12.5659\right]$$

$$= 0.785368.$$

But $\int_0^1 \frac{dx}{1+x^2} = \left[\tan^{-1}x\right]_0^1 = \tan^{-1}1 - \tan^{-1}0 = \pi/4$

In case of simpson's $\frac{1}{3}$ rule $\frac{\pi}{4} = 0.785366$

$\pi = 4(0.785366) \Rightarrow \pi = 3.141464$

In case of simpson's $\frac{3}{8}$ rule $\frac{\pi}{4} = 0.785368$

$\pi = 4(0.785368) \Rightarrow \pi = 3.141472$

Result :- By simpson's $\frac{1}{3}$ rule $= 0.785366$

$\pi = 3.141464$

By simpson's $\frac{3}{8}$ rule $= 0.785368$

$\pi = 3.141472$,,

UNIT - 4

Solution of Simultaneous Linear system of Equations.

Statement of the problem :-

Solve the system of equations $3x+y+2z=3$, $2x-3y-z=-3$, $x+2y+z=4$ by matrix inversion method

Solution :-

Aim :- The system of equation $3x+y+2z=3$, $2x-3y-z=-3$, $x+2y+z=4$. by matrix inversion method.

Formula :-

Procedure :-

The Given equation can be written as

$$\begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$$

$$A \quad X = B$$

To Find A^{-1} :-

Step-① :- $|A| = \begin{vmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{vmatrix}$

$$= 3[-3+2] - 1[2+1] + 2[4+3]$$

$$= 3[-1] - 1[3] + 2[7]$$

$$= -3-3+14$$

$$= 8.$$

$|A| = 8$

step ②

minors of R_1

$$\text{minor of } 3 = \begin{vmatrix} -3 & -1 \\ 2 & 1 \end{vmatrix} = -1$$

$$\text{minor of } 1 = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 3$$

$$\text{minor of } 2 = \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} = 7$$

minors of R_2

$$\text{minor } 2 = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3$$

$$\text{minor } -3 = \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 1$$

$$\text{minor } -1 = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 5$$

minors of R_3

$$\text{minor of } 1 = \begin{vmatrix} 1 & 2 \\ -3 & -1 \end{vmatrix} = 5$$

$$\text{minor of } 2 = \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} = -7$$

$$\text{minor of } 1 = \begin{vmatrix} 3 & 1 \\ 2 & -3 \end{vmatrix} = -11$$

step ③ $$\text{min } A = \begin{bmatrix} -1 & 3 & 7 \\ -3 & 1 & 5 \\ 5 & -7 & -11 \end{bmatrix}$$

$$\text{cof } A = \begin{bmatrix} -1 & -3 & 7 \\ 3 & 1 & -5 \\ 5 & 7 & -11 \end{bmatrix}$$

$$\text{adj } A = (\text{cof } A)^{-1} = \begin{bmatrix} 1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$$

Step - 4

$$X = A^{-1}B$$

$$= \frac{1}{|A|} \operatorname{adj} A \cdot B$$

$$= \frac{1}{8}\begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}\begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} -3 & -9 & 15 \\ \cdot 9 & -3 & -21 \\ 28 & -20 & -44 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 8 \\ 16 \\ -8 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Result :-

$$X = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Statement of the problem:-

Solve the system of equation $5x+2y+z=12$, $x+4y+2z=15$, $x+2y+5z=20$ by G.J Method.

Solution:-

Aim:- The system of equation $5x+2y+z=12$

$$x+4y+2z=15$$

$$x+2y+5z=20$$

Formula:-

procedure:-

$$[AB] \sim \left[\begin{array}{ccc|c} 5 & 2 & 1 & 12 \\ 1 & 4 & 2 & 15 \\ 1 & 2 & 5 & 20 \end{array}\right] R_1 \leftrightarrow R_3$$

$$[AB] \sim \left[\begin{array}{ccc|c} 1 & 2 & 5 & 20 \\ 1 & 4 & 2 & 15 \\ 5 & 2 & 1 & 12 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 5 & 20 \\ 0 & 2 & -3 & -5 \\ 0 & -8 & -24 & -88 \end{array}\right] \begin{array}{l} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - 5R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 5 & 20 \\ 0 & 2 & -3 & -5 \\ 0 & 1 & 3 & 11 \end{array}\right] R_3 \rightarrow \frac{R_3}{-8}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 5 & 20 \\ 0 & 1 & 3 & 11 \\ 0 & 2 & -3 & -5 \end{array}\right] \quad R_2 \leftrightarrow R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 3 & 11 \\ 0 & 0 & -9 & -27 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - 2R_2 \\ R_3 \to R_3 - 2R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 3 & 11 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_3 \to \frac{R_3}{-9}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 + R_3 \\ R_2 \to R_2 - 3R_3 \end{array}$$

$x = 1,\ y = 2,\ z = 3.$

Result:-

$x = 1,$

$y = 2$

$z = 3.$

UNIT – V

Numerical Solutions of ordinary Differential Equations.

Statement of the problem

Solve the differential equation $\frac{dy}{dx} = x+y$ with $y(0)=1$ $x \in [0,1]$ by Taylor Series Expansion to obtain y for $x=0.1$

Solution :-

Aim :- The differential equation $\frac{dy}{dx} = x+y$ with $y(0)=1$ $x \in [0,1]$ by Taylor Series Expansion to obtain y for $x = 0.1$

Formula :-

$$y(x) = 1 + x + x^2 + \frac{x^3}{3} + \frac{x^4}{12} + \frac{x^5}{60} + \cdots$$

Procedure :-

Given $y' = x+y$ $\quad y(0)=1$

$\Rightarrow x_0 = 0,\ y_0 = 1$

$y' = x+y \qquad y'_0 = x_0 + y_0 = 0+1 = 1$

$y'' = 1+y' \qquad y''_0 = 1 + y'_0 = 1+1 = 2$

$y''' = 0+y'' \qquad y'''_0 = y''_0 = 2$

$y^{iv} = y''' \qquad y^{iv}_0 = y'''_0 = 2$

$y^{v} = y^{iv} \qquad y^{v}_0 = y^{iv}_0 = 2$

$$y(x) = y_0 + (x-x_0)y'_0 + \frac{(x-x_0)^2}{2!}y''_0 + \frac{(x-x_0)^3}{3!}y'''_0 + \cdots$$

$$= 1 + (x-0)1 + \frac{(x-0)^2}{2}(2) + \frac{(x-0)^3}{6}(2) + \frac{(x-0)^4}{24}(2) + \frac{(x-0)^5}{120}(2)$$

$$y(x) = 1 + x + x^2 + \frac{x^3}{3} + \frac{x^4}{12} + \frac{x^5}{60} + \cdots \longrightarrow (1)$$

To find $y(0.1)$

put $x = 0.1$ in (1)

$$y(0.1) = 1 + 0.1 + (0.1)^2 + \frac{(0.1)^3}{3} + \frac{(0.1)^4}{12} + \frac{(0.\cancel{1})^5}{60} + \cdots$$

$$= 1 + 0.1 + 0.01 + \frac{0.001}{3} + \frac{0.0001}{12} + \frac{0.00001}{60} + \cdots$$

$$= 1 + 0.1 + 0.01 + 3.333 + 8.3333$$

$$= 1 + 0.1 + 0.01 + \frac{0.001}{3} + \frac{0.0001}{12} + \frac{0.00001}{60}$$

$$= 1 + 0.1 + 0.01 + 0.00033 + 0.00000833 + 0.0000001666$$

$$y(0.1) \simeq 1.110338776$$

$$= 1.11034.$$

Result :-

$$y(0.1) = 1.11034$$

INTEGRAL TRANSFORMS

EQUATIONS

UNIT - II

Application of Laplace transform to Solution of Differential Equation.

ORDINARY DIEFERENTIAL EQUATIONS

INTRODUCTION

The general solution of non-homogenous ordinary differential equation [ODE] or partial differential Equation equals to the sum of the fundamental solution of the corresponding homogenous equation plus The particular solution of the non-homogenous.

The Laplace transform is an Excellent tool for solveng D.E

$$a\frac{d^2y}{dt^2}+b\frac{dy}{dt}+cy=F(t)$$

$$ay''+by'+cy=F(t),,$$

Statement of the problem :-

Solution of ordinary differential Equations with constant coefficients.

Statement of the problem.

Solve $(D^2+1)x = t\cos 2t$, given $x=0, \frac{dx}{dt}=0$ when $t=0$

Solution :-

Aim :- $(D^2+1)x = t\cos 2t$ given $x=0, \frac{dx}{dt}=0$ when $t=0$

Formula :-

Laplace transform both side

Procedure :-

Given Equation is $(D^2+1)x = t\cos 2t$

$$x'' + x = t\cos 2t$$

Taking Laplace transform on both side

$$L(x'') + L(x) = L(t\cos 2t)$$

$$[p^2L(x) - px(0) - x'(0)] + L(x) = \frac{p^2-4}{(p^2+4)^2}$$

given $x(0) = x'(0) = 0$

$$[p^2L(x) - p(0) - (0)] + L(x) = \frac{p^2-4}{(p^2+4)^2}$$

$$(p^2+1)L(x) = \frac{p^2-4}{(p^2+4)^2}$$

$$L(x) = \frac{p^2-4}{(p^2+4)^2(p^2+1)}$$

$$L(x) = \frac{Ap+B}{p^2+1} + \frac{Cp+D}{p^2+4} + \frac{Ep+F}{(p^2+4)^2}$$

$$\frac{p^2-4}{(p^2+4)^2} = \frac{Ap+B(p^2+4)^2+(p+1)(p^2+1)(p^2+4)+Ep+F(p^2+1)}{(p^2+1)(p^2+4)^2}$$

$$p^2-4 = Ap+B\left[p^4+8p^2+16\right]+(Cp+D)\left[p^4+5p^2+4\right]+(Ep+F)\left[p^2+1\right]$$

$$= Ap^5+8Ap^3+16Ap+Bp^4+8Bp^2+16B+Cp^5+5Cp^3+4Cp+Dp^4+$$

$$5Dp^2+4D+Ep^3+Ep+Fp^2+F$$

Now, Equation p^5, p^4, p^3, p^2, p & constant terms.

p^5 term $A+C=0 \Rightarrow C=-A$

p^4 term $B+D=0 \Rightarrow D=-B$

p^3 term $8A+5C+E=0$

p^2 term $8B+5D+F=1$

p term $16A+4C+E=0$

constant $16B+4D+F=-4$

$$\begin{array}{r} 8A+5C+E=0 \\ 16A+4C+E=0 \\ - \quad - \quad - \\ \hline -8A+C=0 \end{array}$$

$$-8A-A=0$$

$$-9A=0$$

$$\boxed{A=0}, \boxed{C=0}$$

$$8A+5C+E=0$$

$$0+0+E=0$$

$$\boxed{E=0}$$

$$\begin{array}{r} 8B+5D+F=1 \\ 16B+4D+F=-4 \\ - \quad - \quad - \quad + \\ \hline -8B+D=5 \end{array}$$

$$-8B - B = 5$$

$$-9B = 5$$

$$\boxed{B = \frac{-5}{9}}, \boxed{D = \frac{5}{9}}$$

$$8B + 5D + F = 1$$

$$8\left(\frac{-5}{9}\right) + 5\left(\frac{5}{9}\right) + F = 1$$

$$\frac{-40}{9} + \frac{25}{9} + F = 1$$

$$1 + \frac{15}{9} = F$$

$$F = \frac{24}{9} \Rightarrow \boxed{F = \frac{8}{3}}$$

$$L(x) = \frac{AP+B}{P^2+1} + \frac{CP+D}{P^2+4} + \frac{EP+F}{(P^2+4)^2}$$

$$= \frac{-5/9}{P^2+1} + \frac{5/9}{P^2+4} + \frac{8/3}{(P^2+4)^2}$$

$$x = -\frac{5}{9}L^{-1}\left(\frac{1}{P^2+1}\right) + \frac{5}{9}L^{-1}\left(\frac{1}{P^2+4}\right) + \frac{8}{3}L^{-1}\left(\frac{1}{(P^2+4)^2}\right)$$

$$= -\frac{5}{9}\sin t + \frac{5}{18}\sin 2t + \frac{8}{3}\cdot\frac{1}{16}\sin 2t - \frac{8}{3}\cdot\frac{1}{8}t\cos 2t$$

$$= -\frac{5}{9}\sin t + \left[\frac{5}{18} + \frac{1}{16}\right]\sin 2t - \frac{1}{3}t\cos 2t$$

$$= -\frac{5}{9}\sin t + \left(\frac{5+3}{18}\right)\sin 2t - \frac{1}{3}t\cos 2t$$

$$= -\frac{5}{9}\sin t + \frac{4}{9}\sin 2t - \frac{1}{3}t\cos 2t$$

$$= -\frac{5}{9}\sin t + \frac{4}{9}\sin 2t - \frac{1}{3}t\cos 2t,,$$

Result :-

$$x = -\frac{5}{9}\sin t + \frac{4}{9}\sin 2t - \frac{1}{3}t\cos 2t$$

UNIT-II

Application of Laplace Transform.

Statement of the problem

Solution of Simultaneous Differential Equation.

Statement of the problem.

Solve $(D^2-3)x-4y=0$, $x+(D^2+1)y=0$, $t>0$ if $x=y=Dy=0$. $Dx=2$.

when $t=0$.

Solution :-

Aim :- $(D^2-3)x-4y=0$, $x+(D^2+1)y=0$, $t>0$ if $x=y=Dy=0$, $Dx=2$

when $t=0$

Formula :-

Laplace transform both side

procedure :-

Given equation is $(D^2-3)x-4y=0$, $x+(D^2+1)y=0$

$(D^2-3)x-4y=0$ $\qquad$ $x+(D^2+1)y=0$

$x''-3x-4y=0$ $\qquad$ $x+y''+y=0$

Taking Laplace transform on both sids.

$L(x'')-3L(x)-4L(y)=0$

$[p^2L(x)-px(0)-x'(0)]-3L(x)-4L(y)=0$

$p^2L(x)-3L(x)-4L(y)=2$

$(p^2-3)L(x)-4L(y)=2$ ——— (1)

$L(x)+L(y'')+L(y)=0$

$L(x)+p^2L(y)-py(0)-y'(0)+L(y)=0$

$L(x)+(p^2+1)L(y)=0$ ——— (2)

Equⁿ ② × (p^2-3)

$$(p^2-3)L(x)+(p^2-3)(p^2+1)L(y)=0$$

$$(p^2-3)L(x)+(p^4+p^2-3p^2-3)L(y)=0$$

$$(p^2-3)L(x)+(p^4-2p^2-3)L(y)=0 \quad \text{—— (3)}$$

Equⁿ ③ - ①

$$(p^4-2p^2-3+4)L(y)=-2$$

$$(p^4-2p^2+1)L(y)=-2$$

$$L(y)=\frac{-2}{p^4-2p^2+1}$$

$$L(y)=\frac{-2}{(p^2-1)^2}$$

$$L(y)=\frac{-2}{[(p-1)(p+1)]^2}=\frac{-2}{(p-1)^2(p+1)^2}$$

$$=\frac{A}{p-1}+\frac{B}{(p-1)^2}+\frac{C}{p+1}+\frac{D}{(p+1)^2}$$

$$-2=A(p-1)(p+1)^2+B(p+1)^2+C(p-1)(p+1)+D(p-1)^2$$

put $p=1$

$$-2=B(1+1)^2$$

$$-2=4B$$

$$\boxed{B=-\frac{1}{2}}$$

put $p=-1$

$$-2=D(-2)^2$$

$$4D=-2$$

$$\boxed{D=\frac{-1}{2}}$$

p^3 term $A+C=0$

$$\boxed{C=-A}$$

p^2 term $A+B-C+D=0$

$$2A-1=0$$

$$\boxed{A=\frac{1}{2}} \quad \boxed{C=\frac{-1}{2}}$$

$$L(y) = \frac{1}{2}\left(\frac{1}{P-1}\right) - \frac{1}{2}\left(\frac{1}{(P-1)^2}\right) - \frac{1}{2}\left(\frac{1}{P+1}\right) - \frac{1}{2}\left(\frac{1}{(P+1)^2}\right)$$

$$y = \frac{1}{2}L^{-1}\left(\frac{1}{P-1}\right) - \frac{1}{2}L^{-1}\left(\frac{1}{(P-1)^2}\right) - \frac{1}{2}L^{-1}\left(\frac{1}{P+1}\right) - \frac{1}{2}L^{-1}\left(\frac{1}{(P+1)^2}\right)$$

$$= \frac{1}{2}e^{t} - \frac{1}{2}te^{t} - \frac{1}{2}e^{-t} - \frac{1}{2}te^{-t}$$

$$= \frac{1}{2}\left(e^{t} - te^{t} - e^{-t} - te^{-t}\right)$$

Eqn ① $\times (P^2+1)L(x)$

$$(P^2+1)(P^2-3)L(x) - 4(P^2+1)L(y) = 2(P^2+1)$$

$$4L(x) + 4(P^2+1)L(y) = 0$$

$$(P^4 - 2P^2 - 3 + 4)L(x) = 2(P^2+1)$$

$$L(x) = \frac{2(P^2+1)}{P^4 - 2P^2 + 1}$$

$$L(x) = \frac{2(P^2+1)}{(P^2-1)^2}$$

$$L(x) = \frac{2(P^2+1)}{[(P+1)(P-1)]^2}$$

$$L(x) = \frac{2(P^2+1)}{(P+1)^2(P-1)^2}$$

$$L(x) = \frac{1}{(P-1)^2} + \frac{1}{(P+1)^2}$$

$$x = L^{-1}\left(\frac{1}{(P-1)^2}\right) + L^{-1}\left(\frac{1}{(P+1)^2}\right)$$

$$x = te^{t} + te^{-t}$$

$$x = t\left(e^{t} + e^{-t}\right)$$

Result :- $x = t(e^{t} + e^{-t})$.

UNIT-III

Application of Laplace Transform to

Integral Equations

Applications of Laplace Transforms To Solutions of Differential equations.

INTRODUCTION

It is used to convert complex differential equation to a simpler from having polynomials it is used to convert derivaties into multiple domain variables and then convert the polynomials backs to the differential Equation using inverse Laplace transform.

Converting the Differential equation to integral equation :-

Statement of the problem :-

converting $y''(t)-3y'(t)+2y(t)=4\sin t$, $y(0)=1$, $y'(0)=2$ into integral equation.

Solution :-

Aim :- $y''(t)-3y'(t)+2y(t)=4\sin t$, $y(0)=1$, $y'(0)=-2$

Formula :- Integrating on both side

Procedure :-

Given equation is $y''(t)-3y'(t)+2y(t)=4\sin t$

integrating on both side from 0 to t

$$\int_0^t y''(u)du-3\int_0^t y'(u)du+2\int_0^t y(u)du=4\int_0^t \sin u\,du$$

$$\left[y'(u)\right]_0^t-3\left[y(u)\right]_0^t+2\int_0^t y(u)du=4\left[-\cos u\right]_0^t$$

$$y'(t)-y'(0)-3\left[y(t)-y(0)\right]+2\int_0^t y(u)du=-4\left[\cos t-\cos 0\right]$$

$$\left[y'(t)-(-2)\right]-3\left[y(t)-1\right]+2\int_0^t y(u)du=-4\left[\cos t-1\right]$$

$$\left[y'(t)+2\right]-3\left[y(t)-1\right]+2\int_0^t y(u)du=-4\left[\cos t-1\right]$$

$$\left[y'(t)+2\right]-3\left[y(t)-1\right]+2\int_0^t y(u)du=4-4\cos t$$

$$y'(t)+2-3y(t)+3+2\int_0^t y(u)du = 4-4\cos t$$

$$y'(t)-3y(t)+5+2\int_0^t y(u)du = 4-4\cos t$$

$$y'(t)-3y(t)+2\int_0^t y(u)du = -1-4\cos t$$

Again integrating from 0 to t we get

$$\int_0^t y'(u)du - 3\int_0^t y(u)du + 2\int_0^t (t-u)y(u)du = -\int_0^t (1+4\cos t)du$$

$$\left[y(u)\right]_0^t - 3\int_0^t y(u)du + 2\int_0^t (t-u)y(u)du = -\left[u+4\sin u\right]_0^t$$

$$\left[y(t)-y(0)\right] + \int_0^t (2(t-u)-3)\,y(u)du = -\left[t+4\sin t\right]$$

$$y(t)-1+\int_0^t (2(t-u)-3)\,y(u)du = -t-4\sin t$$

$$y(t)-1+\int_0^t (2(t-u)-3)\,y(u)du = -t-4\sin t$$

$$y(t)-1+\int_0^t (2(t-u)-3)\,y(u)du = -t-4\sin t$$

$$y(t)+\int_0^t (2(t-u)-3)\,y(u)du = 1-t-4\sin t,$$

Result:- $y(t)+\int_0^t (2(t-u)-3)y(u)du = 1-t-4\sin t.$

Statement of the problem

Applications of Laplace transforms of integral Equations.

Statement of the problem.

Solve the integral equation $f(t) = e^{-t} - 2\int_0^t \cos(t-u) f(u)\,du$.

Solution :-

Aim :- $f(t) = e^{-t} - 2\int_0^t \cos(t-u) f(u)\,du$

Formula :- Laplace transform on both side

procedure :-

Given equation is $f(t) = e^{-t} - 2\int_0^t \cos(t-u) f(u)\,du$

The given eqn can be expressed as $y(t) = 1 - e^{-t} + y(t) * \sin t$

$$y(t) = 1 - e^{-t} + y(t) * \sin t$$

Taking Laplace transform on both side

$$L(y(t)) = L(1) - L(e^{-t}) + L(y(t)) \cdot L(\sin t)$$

$$L(y(t)) = \frac{1}{p} - \frac{1}{p+1} + \frac{1}{p^2+1} L(y(t))$$

$$L(y(t)) - \frac{1}{p^2+1} L(y(t)) = \frac{1}{p} - \frac{1}{p+1}$$

$$\left(1 - \frac{1}{p^2+1}\right) L(y(t)) = \frac{1}{p} - \frac{1}{p+1}$$

$$\left(\frac{p^2+1-1}{p^2+1}\right)L(y(t)) = \frac{p+1-1}{p(p+1)}$$

$$\left(\frac{p^2}{p^2+1}\right)L(y(t)) = \frac{p}{p(p+1)}$$

$$\left(\frac{p^2}{p^2+1}\right)L(y(t)) = \frac{1}{p+1}$$

$$L(y(t)) = \frac{p^2+1}{(p+1)p^3}$$

$$= \frac{A}{p} + \frac{B}{p^2} + \frac{C}{p^3} + \frac{D}{p+1}$$

$$p^2+1 = Ap^2(p+1) + Bp(p+1) + C(p+1) + Dp^3$$

put $p=0$

$1 = C(1)$

$\boxed{C=1}$

put $p=-1$

$2 = D(-1)^3$

$\boxed{D=-2}$

p^3 term $A+D=0$

$A-2=0$

$\boxed{A=2}$

p^2 term $A+B=1$

$2+B=1$

$\boxed{B=-1}$

constant

$B+C=0$

$-1+C=0$

$\boxed{C=1}$

$$L(y(t)) = \frac{2}{p} - \frac{1}{p^2} + \frac{1}{p^3} - \frac{2}{p+1}$$

$$y(t) = 2L^{-1}\left(\frac{1}{p}\right) - L^{-1}\left(\frac{1}{p^2}\right) + L^{-1}\left(\frac{1}{p^3}\right) - 2L^{-1}\left(\frac{1}{p+1}\right)$$

$$= 2 - t - \frac{1}{2}t^2 + 2e^{-t}$$

$$f(t) = e^{-t} - 2e^{-t}t + e^{-t}t^2$$

$$= e^{-t}(t^2 - 2t + 1)$$

Result:- $f(t) = e^{-t}(t-1)^2$